TRINITY COLLEGE

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures | | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

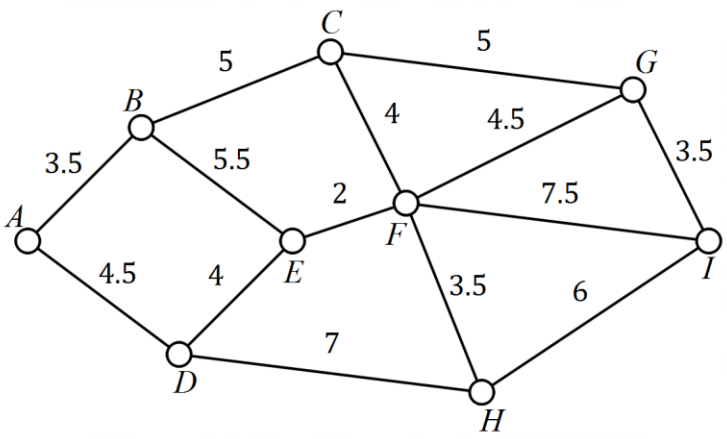
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

The weights on the edges of the graph below are the flight times in hours between adjacent airports, represented by the vertices numbered 1 to 9.



(a) Determine the minimum total flight time between airport A and airport I and state the corresponding route. (2 marks)

Solution
Minimum time is 17 hours. Airports A, B, C, G, I
Specific behaviours
✓ route ✓ minimum time

(b) When planning a journey, a traveller allows 90 minutes at each airport on their route, including the first and last, to allow for check in, transfers, baggage collection and so on. Determine the quickest route for this traveller from airport 1 to airport 9 and state their total journey time. (3 marks)

Solution
Airports A, B, C, G, I: $17 + 5 \times 1.5 = 24.5$ hours. Airports A, D, H, I: $17.5 + 4 \times 1.5 = 23.5$ hours. Quickest route: Airports A, D, H, I. Total journey time is 23.5 hours.
Specific behaviours
✓ indicates at least one correct journey time ✓ correct route ✓ correct journey time

See next page

Question 10 **(7 marks)**

Using the reducing balance method of depreciation, the value of an industrial machine at the end of each year forms a sequence such that the value at the end of its first year of use (year 1) is $\$40\ 800$ and at the end of year 2 is $\$34\ 680$.

The value of the machine in dollars at the end of year n can be modelled by the recurrence relation $V_{n+1} = 0.85V_n,\ V_1 = 40\ 800$.

(a) Show mathematically how to derive the value 0.85 in the recurrence relation from information in the question. (1 mark)

Solution
$34\ 680 \div 40\ 800 = 0.85$
Specific behaviours
✓ shows quotient

(b) Write the rule for the n^{th} term of this sequence. (1 mark)

Solution
$V_n = 40\ 800(0.85)^{n-1}$
Specific behaviours
✓ correct formula

(c) At the end of which year is the machine first valued at less than $\$7\ 500$? Justify your answer. (2 marks)

Solution
At the end of year 12. $V_{11} = 8\ 032.48$ but $V_{12} = 6\ 827.60$.
Specific behaviours
✓ correct year ✓ shows terms either side of $\$7\ 500$

(d) Determine the value of the machine when it was new (at the start of the first year) and hence calculate its total decline in value, to the nearest dollar, over its first five years of use. (3 marks)

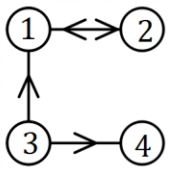
Solution
$V_0 = 40\ 800 \div 0.85 = \$48\ 000$ $V_0 - V_5 = 48\ 000 - 21\ 297.86$ $= \$26\ 702.14$ Total decline in value is $\$26\ 702$.
Specific behaviours
✓ initial value ✓ calculates value after 5 years ✓ total decline, rounded to nearest dollar

See next page

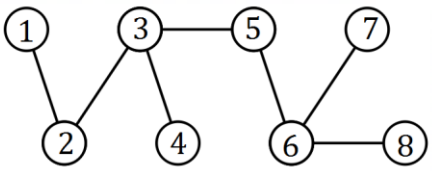
Question 11 **(6 marks)**

(a) Connected graphs G_1 and G_2 are shown at right.

The adjacency matrices for G_1 and G_2 are A_1 and A_2 respectively.



Graph G_1



Graph G_2

(i) Construct matrix A_1. (2 marks)

Solution
$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Specific behaviours
✓ square 4×4 matrix ✓ correct matrix

(ii) Determine the number of entries in A_2 that are 0. (2 marks)

Solution
A_2 will have $8 \times 8 = 64$ entries. There are 7 edges in G_2 and each will lead to two 1's in A_2 and so that will leave $64 - 2 \times 7 = 50$ entries that are 0 in A_2.
Specific behaviours
✓ indicates A_2 will have 64 entries ✓ correct number

(b) The adjacency matrix for graph G_3 is $\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$.

State, with justification, whether G_3 is a simple graph. (2 marks)

Solution
No - there are multiple edges between some vertices (between v_2, v_3 and v_4, v_2).
Specific behaviours
✓ states no with justification ✓ justification

See next page

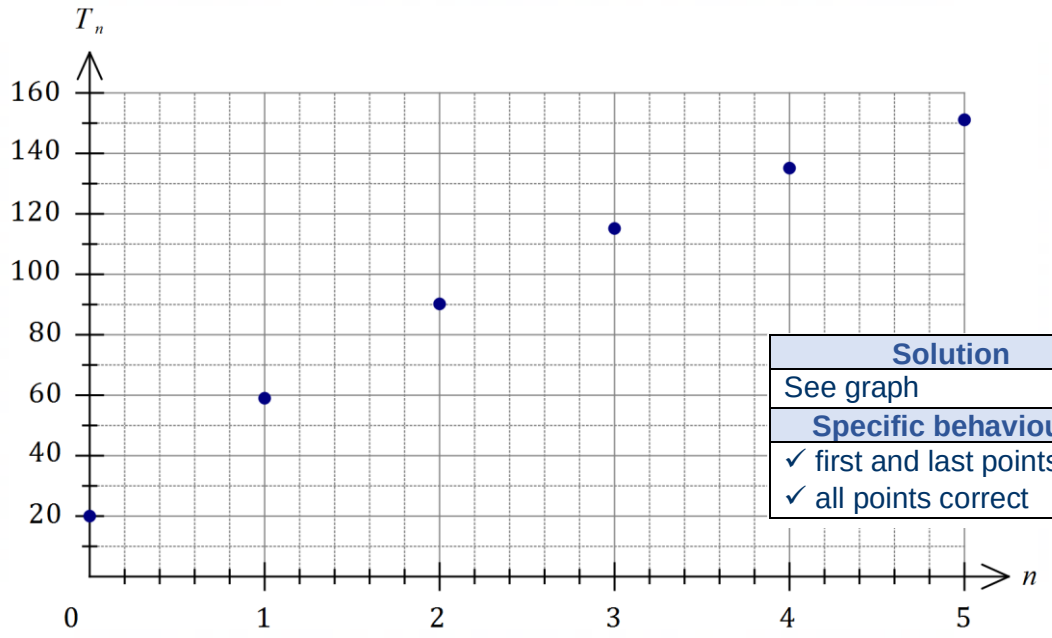
The temperature T_n, in degrees Celsius, of an oven n minutes after being turned on can be modelled by the recurrence relation $T_{n+1} = 0.8T_n + 43,\ T_0 = 20$.

(a) Use the recurrence relation to complete the following table to the nearest degree Celsius. (2 marks)

n	0	1	2	3	4	5
T_n	**20**	**59**	**90**	**115**	**135**	**151**

Solution
See table
Specific behaviours
✓ at least three correct values; ✓ all correct

(b) Plot the temperature of the oven at one minute intervals on the axes below. (2 marks)



Solution
See graph
Specific behaviours
✓ first and last points
✓ all points correct

(c) State the value of n for which the temperature of the oven first exceeds $200\ °C$. (1 mark)

Solution
$n = 12$
Specific behaviours
✓ correct value

(d) Explain how the temperature of the oven changes in the long term. (2 marks)

Solution
The temperature increases become smaller and smaller as the temperature tends towards $215\ °C$.
Specific behaviours
✓ indicates tends to steady state
✓ steady state temperature

See next page

Question 13 **(7 marks)**

The balance of a savings account A_n after n monthly deposits have been made can be modelled by $A_{n+1} = 1.015A_n + 200,\ A_0 = 2\,400$.

(a) Determine

(i) the amount deposited each month. (1 mark)

Solution
$200
Specific behaviours
✓ correct amount

(ii) the annual interest rate of the savings account. (1 mark)

Solution
$0.015 \times 100 \times 12 = 18\%$ p.a.
Specific behaviours
✓ correct rate

(iii) the balance of the savings account after 4 monthly deposits have been made. (1 mark)

Solution
$A_4 = \$3\,365.45$
Specific behaviours
✓ correct amount

After the 240^{th} monthly deposit, no further deposits are made.

(b) Calculate the total interest that the savings account received up to this time. (2 marks)

Solution
$I = A_{240} - 2400 - 240 \times 200$ $= 547\,289.63 - 2\,400 - 48\,000$ $= \$496\,889.63$
Specific behaviours
✓ indicates appropriate method ✓ correct amount

The accumulated balance still attracts interest, compounded monthly at the same rate, and is used to fund an annual perpetuity.

(c) Determine the amount of the annual perpetuity. (2 marks)

Solution
$Q = (1.015^{12} - 1) \times 547\,289.63$ $= \$107\,059.80$
Specific behaviours
✓ indicates appropriate method ✓ correct amount

See next page

Question 14 **(10 marks)**

A factory operates three consecutive eight-hour shifts A, B and C each day. The table below shows the number of workers who turned up late for each shift, together with a three-point moving average m.

Time period t	Day	Shift	Number late	Moving average m
1	Sun	A	47	–
2	Sun	B	35	P
3	Sun	C	41	40
4	Mon	A	44	37
5	Mon	B	26	36
6	Mon	C	38	35
7	Tue	A	41	35
8	Tue	B	26	34
9	Tue	C	Q	–

(a) Briefly describe the purpose of calculating a set of moving averages for a time series. (1 mark)

Solution
To smooth time series data and expose the underlying trend.
Specific behaviours
✓ indicates smoothing

(b) Determine the value of P and the value of Q in the table above. (2 marks)

Solution
$P = (47 + 35 + 41) \div 3 = 41$ $34 = (41 + 26 + Q) \div 3 \Rightarrow Q = 35$
Specific behaviours
✓ value of P ✓ value of Q

(c) Determine the centred six-point moving average for $t = 4$. (2 marks)

Solution
$\left(\frac{47}{2} + 35 + 41 + 44 + 26 + 38 + \frac{41}{2}\right) \div 6 = 38$
Specific behaviours
✓ indicates appropriate method to centre ✓ correct average

See next page

(d) Determine the least-squares line to predict m from t. (2 marks)

Solution
$m = -1.18t + 42.75$
Specific behaviours
✓ slope, to at least 2 dp
✓ intercept, to at least 1 dp
NB Using CAS, first entry for t must be 2.

Two of the seasonal indices for the above time series are shown in the table below.

Shift	A	B	C
Seasonal index	1.19	0.78	

(e) Calculate the seasonal index for shift C. (1 mark)

Solution
$SI_C = 3 - 1.19 - 0.78 = 1.03$
Specific behaviours
✓ value of index

(f) Forecast the number of late workers for the next B shift (on Wednesday), using the least-squares line from (d) and making any necessary seasonal adjustment. (2 marks)

Solution
$t = 11$ $m = -1.18(11) + 42.75$ $= 29.8$ Forecast $29.8 \times 0.78 = 23$ late workers.
Specific behaviours
✓ value using least-squares line
✓ correct forecast, as whole number

See next page

Question 15 **(7 marks)**

On 1 March 2020 Dea started a new job with an annual salary of \$72 000. At that time, the balance of her superannuation fund from previous jobs was \$53 520. Dea's new employer deposits a sum equal to 9.25% of her monthly salary into her fund on the last day of each month.

Interest on the balance of an individual's superannuation fund is added on the last day of each month, just before any deposits are made, and the fund advertises an interest rate of 5.4% per annum.

(a) Determine the balance of Dea's superannuation fund on 1 April 2020. (3 marks)

Solution
Interest multiplier: $1 + 0.054 \div 12 = 1.0045$. Deposit: $72\,000 \div 12 \times 0.0925 = \$555$. New balance: $53\,520 \times 1.0045 + 555 = 53\,520 + 240.84 + 555$ $= \$54\,315.84$
Specific behaviours
✓ interest multiplier (or interest amount) ✓ deposit ✓ correct balance

(b) Write a recursive relation for the balance T_n of Dea's superannuation fund n months after she started her new job. (2 marks)

Solution
$T_{n+1} = T_n \times 1.0045 + 555, \quad T_0 = 53\,520$
Specific behaviours
✓ recursive part ✓ initial term T_0

(c) Calculate the expected increase in the balance of Dea's superannuation fund after she has been in her new job for one year if her circumstances do not change. (2 marks)

Solution
$T_{12} - T_0 = 63\,310.03 - 53\,520.00$ $= \$9\,790.03$
Specific behaviours
✓ correct balance after one year ✓ correct increase

See next page

Question 16 **(8 marks)**

The records of 261 people who were hospitalised with an injury following a road accident have been categorised by road user group and main body region injured in the table below.

	Shoulder	Head	Lower limb	Neck
Motorcyclist	23	7	15	2
Car occupant	49	74	28	63

(a) Determine what percentage of those hospitalised were motorcyclists. (1 mark)

Solution
$23 + 7 + 15 + 2 = 47$ $47 \div 261 = 18\%$
Specific behaviours
✓ correct percentage

(b) State the most common main body region injured by motorcyclists and what percentage of motorcyclists had this body region recorded as their main injury. (2 marks)

Solution
Shoulder, for $23 \div 47 = 49\%$ of motorcyclists.
Specific behaviours
✓ correct region ✓ correct percentage

(c) Complete the following table of row percentages, rounding to the nearest whole number. (3 marks)

(%)	Shoulder	Head	Lower limb	Neck
Motorcyclist	49	15	32	4
Car occupant	23	35	13	29

Solution
See table
Specific behaviours
✓ at least two correct entries; ✓ one row correct; ✓ all entries correct

(d) Do the records suggest the presence of an association between the categorical variables? Justify your answer. (2 marks)

Solution
Yes, because the percentages in each column (across categories) are quite different. For example, only 4% of motorcyclists had neck as their main injury compared to 29% of car occupants.
Specific behaviours
✓ states yes to association ✓ observes general differences or uses specific example

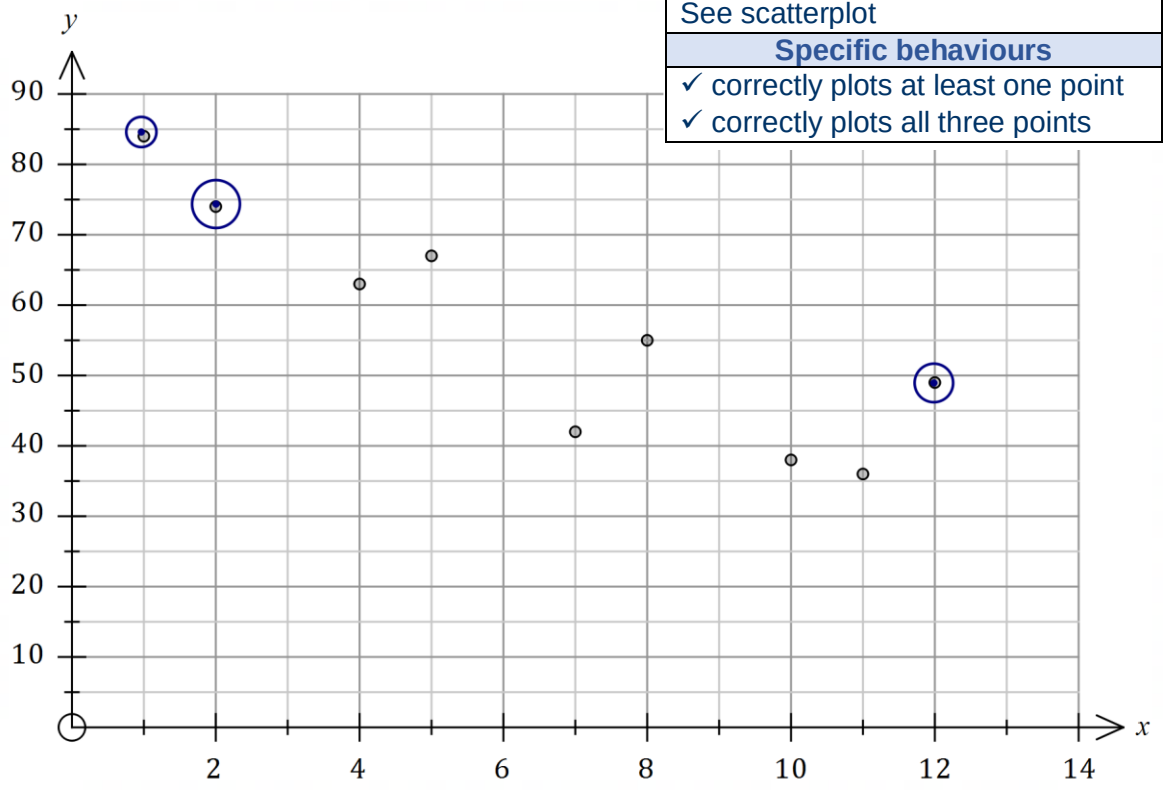
See next page

Question 17 **(11 marks)**

The table below shows the percentage of all trips made using a bicycle x and a car y for nine countries. The correlation coefficient for the set of data is -0.89.

Country	A	B	C	D	E	F	G	H	I
Bicycle trips, x (%)	4	1	12	5	8	2	11	10	7
Car trips, y (%)	63	84	49	67	55	74	36	38	42

(a) Add the three missing points to the scatterplot below. (2 marks)



Solution
See scatterplot
Specific behaviours
✓ correctly plots at least one point
✓ correctly plots all three points

(b) A journalist discussed the dataset in an article with the headline "Decreasing bicycle use causes increase use of cars". Comment on their choice of headline. (2 marks)

Solution
The headline is misleading as the observed negative association between the variables does not necessarily mean that a change in car use is caused by a change in bicycle use.
Specific behaviours
✓ indicates that headline is misleading/inappropriate/etc
✓ comment(s) related to causation

See next page

(c) Determine the equation of the least-squares line with x as the explanatory variable. (2 marks)

Solution
$y = -3.79x + 81.7$
Specific behaviours
✓ slope to at least 2 dp
✓ intercept to at least 1 dp

(d) In the context of the question, interpret

(i) the intercept of the least-squares line. (1 mark)

Solution
In a country where no trips were made by bicycle, close to 82% of trips would be made using a car.
Specific behaviours
✓ interpretation using intercept

(ii) the slope of the least-squares line. (1 mark)

Solution
For every 1% increase in the percentage of trips made by bicycle, there is an observed decrease close to 4% in the percentage of trips made by car.
Specific behaviours
✓ reasonable interpretation of negative gradient

(e) In country K, 15% of all trips are made by bicycle. Predict the percentage of trips made using a car in this country and discuss factors related to the use of the fitted line that affect your confidence in this prediction. (3 marks)

Solution
$y = -3.79(15) + 81.7 \approx 25\%$ The strong correlation coefficient of -0.89 would usually lead to high confidence but due to the large amount of extrapolation, confidence in the prediction is low.
Specific behaviours
✓ correct prediction (that rounds to given value) ✓ discusses strong correlation ✓ discusses extrapolation

See next page

Question 18 **(9 marks)**

Data for the number of regional home sales per quarter, rounded to the nearest hundred homes, is shown in the table below.

t	Year	Quarter	Sales (h), in hundreds	Quarterly mean	Sales as percentage of quarterly mean
1	2011	1	56	A	92.9
2		2	61		101.2
3		3	59		97.9
4		4	65		107.9
5	2012	1	64	74.0	86.5
6		2	76		102.7
7		3	74		100
8		4	82		B
9	2013	1	79	83.8	94.3
10		2	91		108.7
11		3	80		95.5
12		4	C		101.5

(a) Describe the trend and seasonality of this data. (2 marks)

Solution
There is an increasing trend. In each year, sales tend to be lowest in Q1, increase in Q2, decrease in Q3 and end highest in Q4.
Specific behaviours
✓ describes trend ✓ describes seasonality

See next page

(b) Calculate the value of A, the value of B and the value of C in the table. (4 marks)

Solution
$A = (56 + 61 + 59 + 65) \div 4 = 60.25$ $B = 82 \div 74 = 110.8\%$ $C \div 83.77 = 101.5 \Rightarrow C = 85$ or $(79 + 91 + 80 + C) \div 4 = 83.77 \Rightarrow C = 85$
Specific behaviours
✓ value of A ✓ value of B ✓ value of C

(c) Determine the deseasonalised number of home sales in the region in the first quarter of 2013. (3 marks)

Solution
$SI_{Q1} = (0.929 + 0.865 + 0.943) \div 3 = 0.9125$ $79 \div 0.9125 = 86.6$ The deseasonalised number of sales is 8660 homes.
Specific behaviours
✓ calculates seasonal index ✓ divides sales by index ✓ value that rounds to 8700 and allows for hundreds

(d) Forecast the actual number of home sales in the region for the first quarter of 2014 given that the least-squares trend line for the deseasonalised data indicates that the number of home sales to be 8 880 at that time. (1 mark)

Solution
$h = 8\,880 \times 0.9125 = 8\,100$
Specific behaviours
✓ correct value that rounds to 8 100

See next page

Question 19 **(7 marks)**

A company took out a business loan of $\$125\ 000$ at an interest rate of 8.76% per annum and made monthly repayments of $\$5\ 700$. The first few rows of a spreadsheet used by the company to track the loan balance is shown below.

Month, n	Balance at start of month n	Interest	Repayment	Balance carried forward
1	125 000.00	912.50	5 700.00	120 212.50
2	120 212.50	877.55	5 700.00	115 390.05
3	115 390.05	A	5 700.00	B

(a) Determine the value of A and the value of B in the spreadsheet. (2 marks)

Solution
$A = 115\ 390.05 \times 0.0876 \div 12 = 842.35$ $B = 115\ 390.05 + 842.35 - 5700 = 110\ 532.40$
Specific behaviours
✓ value of A ✓ value of B

A recurrence relation of the form $T_{n+1} = rT_n - d,\ T_1 = a$ can be used to model the balance of the loan at the start of month n.

(b) Determine the value of each of the constants r, d and a in the recurrence relation. (2 marks)

Solution
$r = 1 + (8.76 \div 12 \div 100) = 1.0073$ $d = 5\ 700, \quad a = 125\ 000$
Specific behaviours
✓ value of r (to at least 5 dp) ✓ value of d and value of a

(c) Using a financial calculator, or otherwise, state

(i) the balance of the loan after 9 repayments. (1 mark)

Solution
$\$80\ 632.68$
Specific behaviours
✓ correct value

(ii) the number of repayments required to repay the loan. (1 mark)

Solution
24 months.
Specific behaviours
✓ correct value

(iii) the minimum monthly repayment for the full amount of the loan to be repaid at the same interest rate in 12 equal repayments. (1 mark)

Solution
$\$10\ 917.53$
Specific behaviours
✓ correct value

See next page

Question 20 **(7 marks)**

The number of minutes that Ethan spends each day on homework can be modelled by sequence A. Some consecutive terms of sequence A are shown in the following table.

n	4	5	6	7	8
A_n	55	49	43	37	31

(a) State the name given to this type of sequence and determine A_1, the first term of the sequence. (2 marks)

Solution

Arithmetic sequence.

$55 = a + 3(-6) \Rightarrow a = A_1 = 73$

Specific behaviours

✓ name of sequence
✓ first term

(b) Determine a rule for the n^{th} term of sequence A in the form $A_n = m \times n + c$, where m and c are both constants. (2 marks)

Solution

$$A_n = 73 + (n-1)(-6)$$
$$= 73 - 6n + 6$$
$$= -6n + 79$$

Specific behaviours

✓ uses rule on formula sheet
✓ simplifies and states in required form

The n^{th} term of sequence B is B_n, so that $B_4 = A_{17}$ and $B_{10} = A_{12}$.

(c) Determine B_{14}. (3 marks)

Solution

$B_4 = A_{17} = -23, \quad B_{10} = A_{12} = 7$

$6d = 7 - (-23) = 30 \Rightarrow d = 5$

$B_{14} = B_{10} + 4d = 7 + 4(5) = 27$

Specific behaviours

✓ two values of sequence B
✓ common difference d
✓ correct term

See next page

Question 21 **(7 marks)**

A person has a credit card account with an outstanding debt of $\$2\,883$ and the card provider charges interest at a rate of 14.49% per annum compounded daily.

(a) Determine their card debt in 30 days' time if the card is not used for any more purchases and no repayments are made. (2 marks)

Solution
$F = 2\,883\left(1+\frac{14.49}{100\times 365}\right)^{30}$ $= \$2\,917.53$
Specific behaviours
✓ indicates method (possibly a financial calculator) ✓ correct debt

The person can pay off their card debt using an unsecured loan from their bank at an interest rate of 14.6% compounded monthly.

(b) Use effective interest rates to determine, with reasoning, whether the unsecured loan would be a better option for this person. (3 marks)

Solution
Card: $E = 15.59\%$ p.a. Loan: $E = 15.62\%$ p.a. The loan is not a better option as the effective interest rate is higher.
Specific behaviours
✓ one correct rate (to at least 2 dp) ✓ both correct rates ✓ explains why loan is not a better option

The person chose to pay off their card debt in full by taking out a two year secured loan for $\$2\,883$ from a lender who compounds interest quarterly. At the end of this time, the person must repay the principal and interest, a sum of $\$3\,417.84$.

(c) Determine the interest rate charged by this lender. (2 marks)

Solution
$3\,417.84 = 2\,883\left(1+\frac{r}{100\times 4}\right)^{8}$ $r = 8.6\%$
Specific behaviours
✓ indicates method (possibly a financial calculator) ✓ correct rate

End of questions